SURF's up

An outline of an innovative framework for teaching mental computation to students in the early years of schooling

James Russo
Monash University
<james.russo@monash.edu>

In this article James Russo presents the Strategies, Understanding, Reading and Fast Facts Framework (SURF) for mental computation. He explains how this framework can be used to deepen mathematical understanding and build mental flexibility.

A brief history of the framework

Identifying a need

Mental computation strategies have become increasingly emphasised in the teaching of mathematics, because they help to build mental flexibility, efficient approaches to solving maths problems and mathematical fluency (Beishuizen & Anghileri, 1998). Despite its recent prominence, the teaching of mental computation strategies appears to lack a clear organisational framework.

There certainly have been texts developed which outline the strategies that may be introduced at various stages of a student's mathematical development and provide explanations for the teaching of these strategies (e.g., *Teaching Mental Calculation Strategies*, developed by the Qualifications and Curriculum Authority, QCA, 1999). However, these texts tend to organise strategies in a long linear sequence, without much emphasis on how they build on or relate to each other, or how they connect to different mathematical goals.

Daily Café Framework

Over the past several years, many Australian schools have successfully implemented the Daily CAFE approach for reading (Bouchey & Moser, 2009). One of the major strengths of Daily CAFE is its organising framework. There are four reading goals comprising the acronym CAFE: Comprehension, Accuracy, Fluency and Expand Vocabulary. The idea is that these four goals of reading are relevant to readers of all levels and that students will focus on working towards one particular goal in a given reading lesson (or series of lessons). In order to help students work towards achieving a given reading goal, a number of strategies are introduced (Bouchey & Moser, 2009).

In general, the strategies are introduced in order of sophistication, with the least sophisticated (but by no means the least important) strategies introduced first. For example, to improve Comprehension, which is defined as understanding what you read, the reader must initially practise the strategies Check for Understanding and Back-up and Re-read. Having mastered these rudimentary strategies, more complex strategies may be introduced, such as *Connecting Text to Prior Knowledge and Asking Questions While Reading* (Bouchey & Moser, 2009).

Developing a framework to support mental computation

A small-to-medium size (260 student enrolments) public primary school in the Dandenong Ranges region of Melbourne has had some success with implementing Daily CAFE as a whole of school approach to reading since 2011. With a small grant from the school council, it was decided to explore the feasibility of developing a similar framework for organising and managing the teaching of mental mathematics, specifically mental computation, at the school.

The primary purpose behind developing an organising framework for mental computation

was that, in a similar manner to CAFE, such a framework could be displayed prominently in the classroom, thus creating a bridge between teacher knowledge and student understanding. Specifically, such a framework was intended both to be accessible to students and to offer a means of assisting teachers to structure mental computation instruction. In this manner, it could create shared dialogue between students and teachers, and facilitate the construction of a common language around mathematics.

In addition, such a framework was anticipated to provide a number of other related benefits, including:

- to refine and better focus the teaching of mental computation strategies at the school;
- to develop stronger links between what is being taught and the current learning needs of individual students;
- to ensure that teachers have a shared understanding of particular mental computation strategies and are approaching the teaching of these strategies in a consistent manner;
- to place more emphasis in the curriculum on meta-cognition as it relates to mental computation strategies; and
- to place more emphasis in the curriculum on the acquisition and understanding of mathematical language.

It was from these beginnings that the idea of SURF Maths was born. The framework is currently being utilised at the school with students in Foundation, Grade 1 and Grade 2, and there is an intention to extend its application to students in Grades 3 and 4 in the coming years. The remainder of the article is dedicated to providing an overview of the framework.

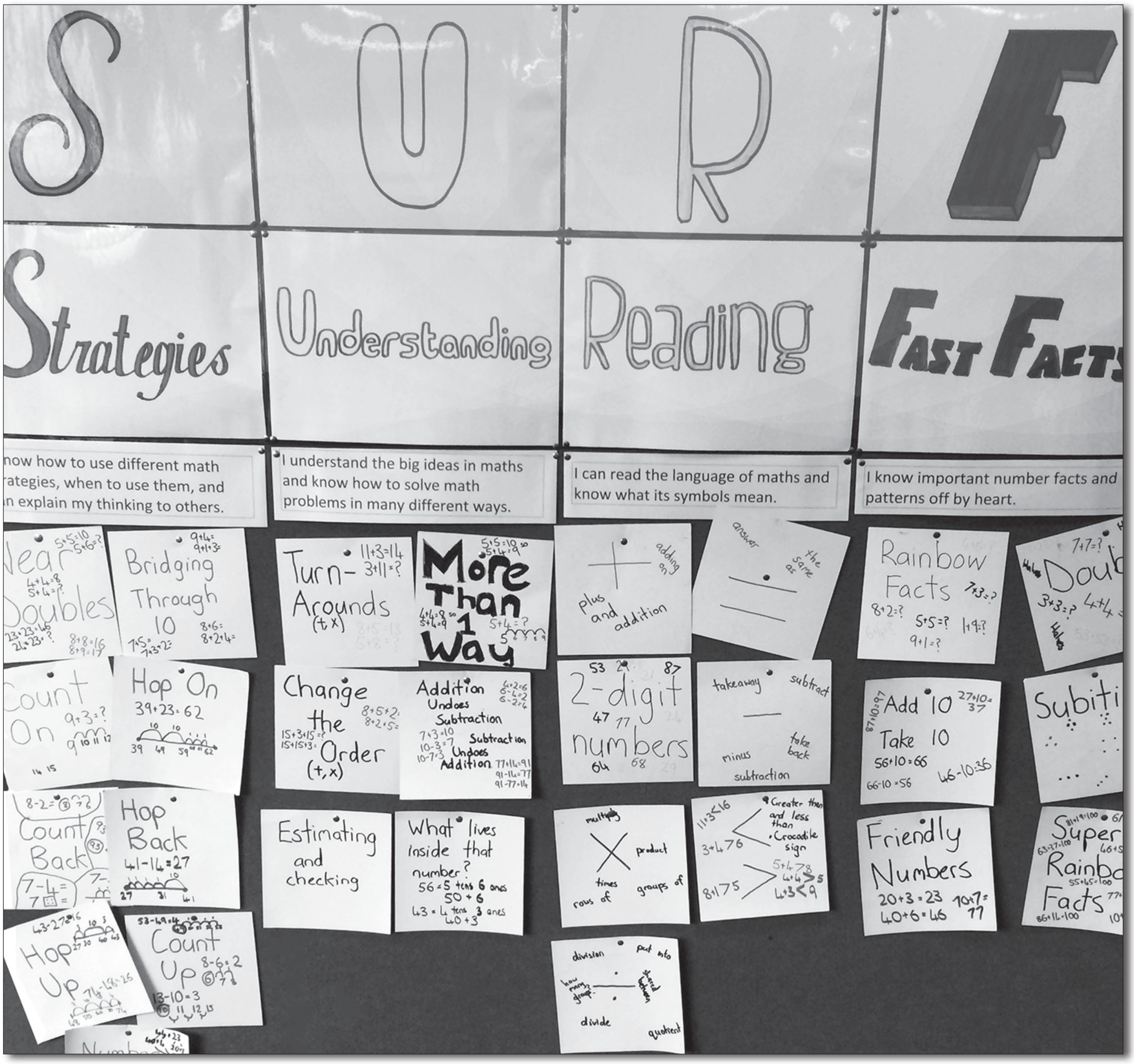

Figure 1: SURF

Overview of SURF Maths: Let's go SURFing

SURF is an organising framework for the teaching of mental computation. In a similar manner to Daily CAFE, SURF Maths is an acronym that stands for Strategies, Understanding, Reading and Fast Facts. The meaning of these terms in the context of the SURF framework is as follows:

- **Strategies:** "I know how to use different maths strategies, when to use them and can explain my thinking to others."
- **Understanding**: "I understand important ideas in maths and know how to solve maths problems in many different ways."
- **Reading:** "I can read the language of maths and know what its symbols mean."
- **Fast Facts:** "I know important number facts and patterns off by heart."

Strategies, Understanding, Reading and Fast Facts can be viewed as the four goals of mental computation. Again, like the CAFE reading framework, which provides a comprehensive list of strategies to support the teaching of its four goals, the SURF framework is supported by 'teaching points' that can be mapped onto each of its goals:

- **Strategies:** The teaching of individual 'strategies' is linked to the overall strategies goal.
- **Understanding:** The teaching of a series of 'ideas' in mathematics is linked to the understanding goal.
- **Reading:** The teaching of mathematical 'symbols' and key mathematical terminology is linked to the goal of being able to read mathematics.
- **Fast Facts:** The teaching of 'facts' is linked to the goal of being able to recall number facts quickly.

What distinguishes SURF from other mental computation frameworks?

At the outset, it is important to emphasise that the majority of the potential content of SURF is available in other mathematical texts (e.g., QCA, 1999), in particular, all of the Strategies and Fast Facts, which comprise most of mental computation as it is currently understood. The strength of the SURF framework is that it makes further explicit the notion that mental computation is about deepening mathematical understanding and building mental flexibility.

For example, one of the key teaching points introduced under the Understanding goal is the idea that there is 'More Than One Way' of solving an arithmetic problem. If faced with the problem 6 + 5 = ?, some students will choose to employ the near doubles strategy (5 + 5 + 1 = 11 or 6 + 6 – 1 = 11), other students will bridge through ten (i.e., 6 + 4 + 1 = 11), whilst other students will likely count on (6… 7, 8, 9, 10, 11). Exploring this diversity of responses can stimulate rich mathematical discussions and lead to the realisation that some approaches are likely to be more efficient than others in particular contexts.

In addition, unlike other mental computation frameworks (e.g., Hartnett, 2008), key concepts within SURF are organised horizontally, in order to make mental computation both more 'teachable' and 'learnable'. For example, SURF makes a distinction between being able to rapidly recall important number patterns (Fast Facts), and manipulating this information to solve related number problems (Strategies).

Specifically, Doubles Facts (e.g., 6 + 6 = 12) and Rainbow Facts (7 + 3 = 10) are both categorised as Fast Facts, whilst near Doubles (6 + 7 = 6 + 6 + 1) and Bridging Through Ten (7 + 4 = 7 + 3 + 1) are both categorised as Strategies.

Potential advantages of the SURF acronym

There are many possible advantages to employing the SURF acronym in a classroom context. Some of these are listed below:

- In a similar manner to the CAFE framework for reading, each of the key terms captures an important goal of mental computation;
- It keeps the number of goals highly manageable (i.e., four goals), allowing these goals to be held in mind by both teachers and students;
- It provides a distinct visual image, which can be used to energise classroom displays for mental computation; and
- Surf(ing) also serves as a powerful metaphor for mental computation. Just like surfing, mental computation requires you to concentrate, make decisions quickly, be flexible and take risks. Students have to have trust in themselves and the strategies they have learnt in order to put it all together. Like surfing, learning mental computation takes considerable practice; however once you begin

to develop some competence and confidence, learning becomes a lot of fun.

Summary of the four goals of SURF

Strategies: "I know how to use different maths strategies, when to use them and can explain my thinking to others"

The Strategies goal is primarily about the learning and application of derived mathematical strategies. By derived strategies, we are referring to all strategies which we do not expect students to recall using direct retrieval. Applying a strategy often involves 'doing something' to our directly retrievable knowledge. Within the SURF framework, we simply refer to these derived strategies as 'strategies', and refer to important patterns we expect students to master through direct retrieval as 'facts'. For example, students, having learnt their 'Doubles Facts', are in a position to apply this knowledge to the problem 8 + 7 by using the 'Near Doubles Strategy' (e.g., 8 + 8 – 1 or 7 + 7 + 1). Although the application of this strategy follows a structured, reliable process, it is not necessary or even advisable to attempt to completely memorise by rote all our near doubles facts if we understand when and how to apply the near doubles strategy.

Some potential Strategies for introduction in a Foundation to Grade 2 classroom, and their related Fast Facts, are included in Table 1. The order in which they are presented is a suggested sequence of introduction, with the less complex strategies being listed first.

Table 1: Strategies and related Fast Facts for students in Foundation to Grade 2

Strategy	Related Fast Fact(s)	Example
Count On	Counting forwards to 20 (and then 100)	7 + 2 = 9, because: 7, 8, 9
Count Back	Counting backwards from 20 (and then 100)	7 – 2 = 5, because: 7, 6 , 5
Count Up	Counting forwards to 20 (and then 100)	16 – 14 = 2, because: 14, 15, 16
Near Doubles	Doubles	6 + 5 = 11, because: 5 + 5 = 10 and 6 – 5 = 1, and 10 + 1 = 11
Bridging Through 10	Rainbow facts (i.e., addition facts which equal 10, such as 3 + 7 and 8 + 2)	9 + 7 = 16, because: 9 + 1 = 10, and 7 – 1 = 6, and 10 + 6 = 16
Double doubles	Doubles	4 groups of 6 = 24 because: Double 6 = 12 and Double 12 = 24
Hop On (count on by 10s and then by 1s)	Counting forwards to 100 Add 10 (e.g., 36 + 10 = 46; 62 + 10 = 72)	21 + 33 = 54, because: 33, 43, 53 and 54.
Hop Back (count back by 10s and then by 1s)	Counting backwards to 100 Subtract 10 (e.g., 23 – 10 = 13; 47 – 10 = 37)	54 – 21 = 33, because: 54, 44, 34 and 33.
Number splitting* (partition into 10s and 1s and then add)	Friendly numbers (e.g., 50 + 6 = 56; 20 + 3 = 23)	35 + 27 = 62 because: 35 = 30 + 5; 27 = 20 + 7 and 30 + 20 = 50; 5 + 7 = 12 and 50 + 12 or 50 + 10 + 2 = 62
Difference method* (begin at the smaller number and bridge to the next multiple of 10, count in multiples of 10 and use friendly numbers until you reach the larger number).	Rainbow facts Skip counting by 10's Friendly numbers	66 – 28 = 38, because: 28 + 2 = 30; 30+ 30 = 60 and 60 + 6 = 66. The total difference between the two numbers is 38 (2 + 30 + 6).

*These strategies also require the integration of more rudimentary strategies.

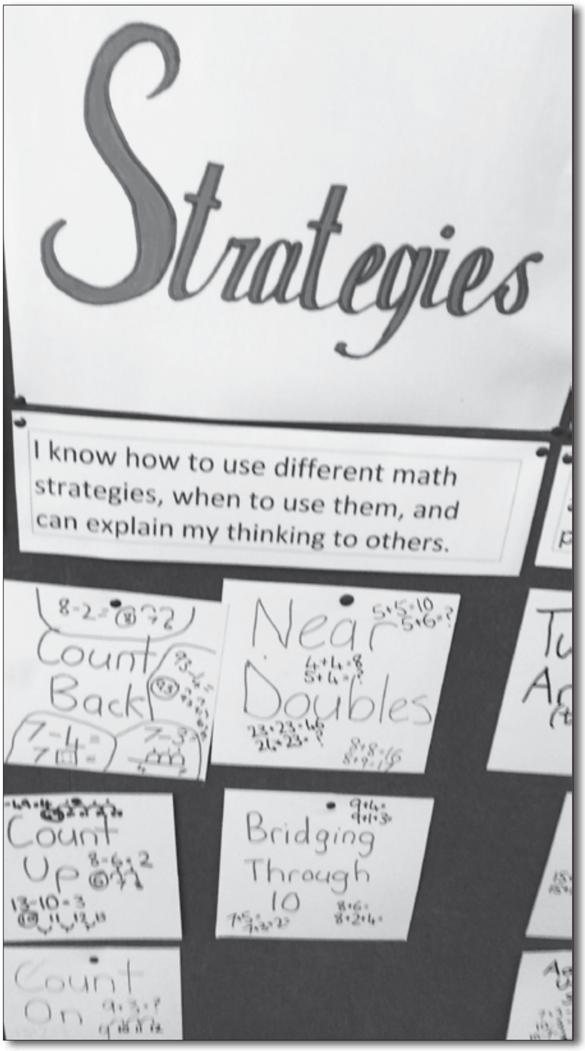

Figure 2: Strategies

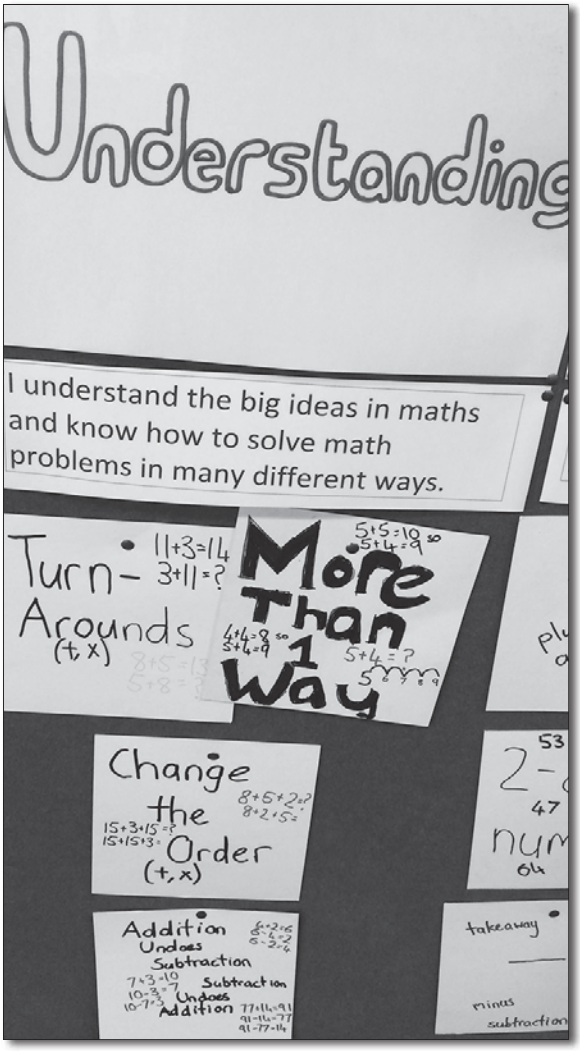

Figure 3: Understanding

Understanding: "I understand the big ideas in maths and know how to solve maths problems in many different ways"

The Understanding goal is all about knowing how mathematics, particularly number and algebra, works. A key part of the Understanding goal is to know what factual knowledge to draw upon in a given context, and which strategy, or set of strategies, to apply to this knowledge in order to answer the question correctly. It also relates to the fundamental properties which govern number operations, such as the inverse relationship between addition and subtraction. The Understanding goal is broader than the other goals, encompassing a larger variety of mental functions. It is focussed around meta-cognitive elements, that is, our capacity to think about our own thinking. It involves knowing how to answer the question and what your answer means. For example, we learn and apply derived mental computation strategies because they are efficient.

However, the other important aspect of learning a broad repertoire of strategies is that it allows us to check if our initial answer is correct. This cross-checking is a vital component of building mental flexibility, and, consequently, the concept of Estimating and Checking is a key idea introduced under the Understanding goal.

Moreover, in order to check we are correct, it is desirable to have knowledge of a variety of derived strategies that can be applied appropriately to any one question. Therefore, as discussed earlier, another important idea introduced to students early on under the Understanding goal is the notion that there is 'More Than One Way'. More Than One Way means that there are a number of ways of solving problems, and students should choose between these strategies based on contextual efficiency, and their individual strengths and preferences as learners.

Key ideas that could be introduced under the Understanding goal, listed in order of a possible sequence of introduction include:

- Turnaround Facts (commutative property)
- More Than One Way
- Change the order (associative property)
- Partitioning (into10s and 1s): what lives inside a number?
- Addition undoes subtraction; subtraction undoes addition
- Fractions
- Estimating and checking

Reading: "I can read the language of maths and know what its symbols mean"

Mathematics has its own abstract language that needs to be learnt by students. The Reading goal within SURF is about students' abilities to understand and apply these abstract codes, as well as an understanding of how these symbols translate into everyday language and vice versa.

At the lower primary level, the Reading goal involves recognising and interpreting the basic operators, equality and inequality signs, and numbers.

Reading is clearly connected to the mathematical goal of Understanding, and in some ways, could be viewed as a subset of Understanding. The advantage of specifying Reading as a separate goal is that it ensures sufficient emphasis is placed within the curriculum on being able to read, interpret and apply the language of mathematics. For example, when confronted with a worded problem, the Reading goal encapsulates the objective of students being able to extract the relevant mathematics from the question. Likewise, the Reading goal also represents the knowledge that a variety of different real world contexts can be represented by the same abstract number sentence (3 + 5 = 8).

Some of the symbols introduced under the Reading goal, again presented in a potential

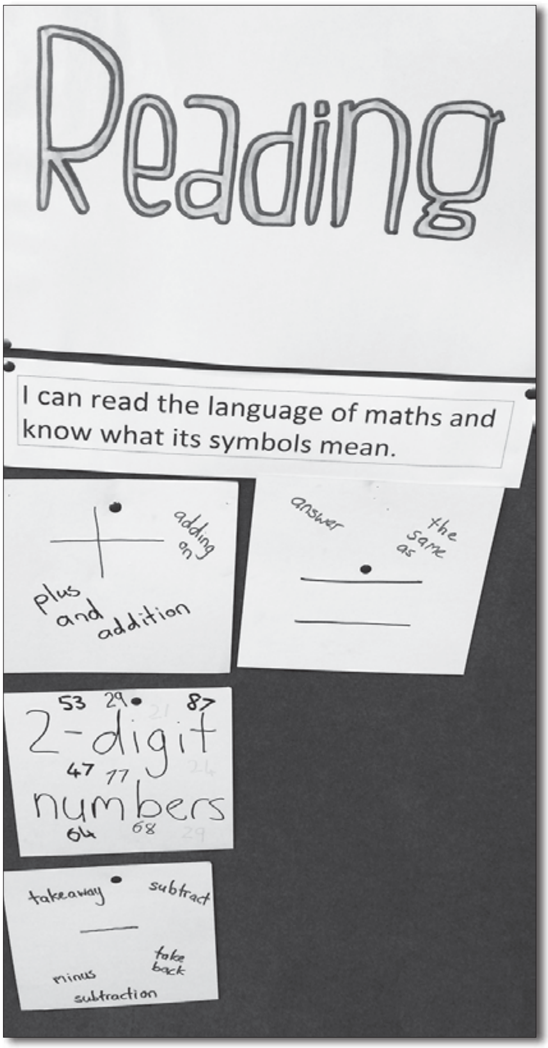

Figure 4: Reading

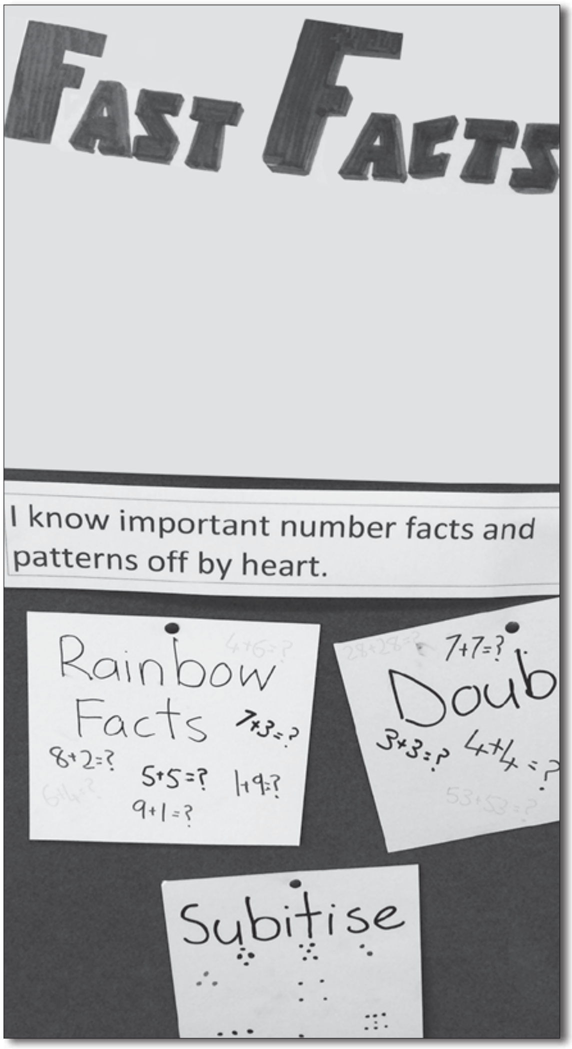

Figure 5: Fast Facts

teaching sequence, may include: 1 digit numbers; 2 digit numbers; +, –, =, > and <; 3 digit numbers; ×, ÷; 4 digit numbers; ∞. The idea is that the classroom teacher will comprehensively annotate these symbols when they are introduced, unpacking the relevant English language terminology captured by these abstract mathematical terms.

Fast Facts: "I know important mathematical patterns off by heart, and can recall facts quickly"

In contrast to Strategies, the Fast Facts goal is primarily about learning simple mathematical patterns so that they can be accurately retrieved without computation. The other component of Fast Facts is building students mathematical processing speed. Both these objectives are consistent with reducing the load on a student's working memory, allowing them to focus on broader aspects of a problem (Cowan, 2003).

Once students have mastered rudimentary, related mathematical constructs such as one-to-one correspondence, cardinality and conservation of number, it is appropriate to introduce basic number patterns, such as 'rainbow facts' (i.e., number bonds equalling ten), counting by 2s or double patterns. Students can then begin to absorb these number patterns, and to gain fluency in the associated number facts. The learning of such patterns and their recall when prompted is what Fast Facts refers to within SURF.

Some Fast Facts that may be introduced at the Foundation to Grade 2 level, again presented in a possible teaching sequence, are listed below. It is worth noting that the introduction of a particular Fast Fact (e.g., Counting forwards to 20/100) needs to occur before the related Strategy (e.g., Count on) is introduced (see Table 1).

- Subitising
- Counting forwards to 20 (and then 100)
- Counting backwards from 20 (and then 100)
- Rainbow facts (i.e., number bonds equalling 10)
- Doubles and halves
- Friendly numbers (i.e., adding a multiple of ten to a single digit number, for example, 50 + 7 = 57)
- Adding 10 and subtracting 10
- Skip counting by 2s, 3s, 5s and 10s.

Where to from here?

Our initial experience in our school with the SURF Maths framework has been extremely positive, and a number of the anticipated benefits outlined earlier in the article are beginning to be realised. At least three other schools in our region have indicated an interest in adopting the framework to support the teaching of mental computation in 2015. At the conclusion of 2015, it is the author's intention to undertake a formative evaluation of the SURF framework, both to capture what has been learnt so far and to further refine the concept. If any other schools or organisations are interested in using the framework, they should contact the author directly via email. Alternatively, readers can access the website www.surfmaths.com, which provides activities, lesson plans and other free resources to help teachers get started with SURF in their own classrooms.

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